Добавка

8. Положительные числа x, y, z, и w таковы, что

$$x^2 + y^2 - \frac{xy}{2} = w^2 + z^2 + \frac{wz}{2} = 36$$

$$xz + yw = 30.$$

Найдите максимальное значение $(xy + wz)^2$.

9. Let a, b, c be positive numbers with $\sqrt{a} + \sqrt{b} + \sqrt{c} = \frac{\sqrt{3}}{2}$. Prove that the system of equations

$$\sqrt{y-a} + \sqrt{z-a} = 1$$

$$\sqrt{z-b} + \sqrt{x-b} = 1$$

$$\sqrt{x-c} + \sqrt{y-c} = 1$$

has exactly one solution (x, y, z) in real numbers.

10. Given positive real numbers a, b, c, d that satisfy equalities

$$a^2 + d^2 - ad = b^2 + c^2 + bc \text{ and } a^2 + b^2 = c^2 + d^2$$

find all possible values of the expression $\frac{ab+cd}{ad+bc}$.

11. Find all ordered triples (x, y, z) of real numbers such that

$$5\left(x + \frac{1}{x}\right) = 12\left(y + \frac{1}{y}\right) = 13\left(z + \frac{1}{z}\right),$$

and

$$xy + yz + zy = 1.$$

12. Given $x, y, z \in (0, 1)$ satisfying that $\sqrt{\frac{1-x}{yz}} + \sqrt{\frac{1-y}{xz}} + \sqrt{\frac{1-z}{xy}} = 2$. Find the maximum value of xyz.